MODEL PREDICTIVE CONTROL FORMULATION: A REVIEW WITH FOCUS ON HYBRID GEOTABS BUILDINGS

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ABSTRACT

Model predictive control (MPC) has been demonstrated to be a potential method for optimal control applied to building energy systems that provides both energy savings and enhanced thermal comfort compared to the traditional rule-based controllers (RBC). Nevertheless, an appropriate and correct formulation of the optimization problem is necessary to achieve this improved controller performance. One of the key aspects of this formulation is the objective/cost function, which usually consists of multiple objectives, typically energy use and thermal discomfort. Depending on the objective formulation, the trade-off between energy savings and occupants’ thermal comfort will be affected. Furthermore, the objective function should match the final user’s preferences, e.g. a “greener” user will prefer to sacrifice some money to reduce the building’s carbon footprint. To this end knowledge of the primary energy sources used allows converting energy use into CO₂ emissions, which could represent another objective to be minimized. When smart meters are available higher economic benefits can be realized by including dynamic prices in the objective function. Moreover, some MPC formulations tend to have a non-robust (bang-bang) behavior which can be avoided by modifying the objective formulation. Furthermore, thermal ground balance (which affects geothermal heat pump performance) can be guaranteed by including a penalization for heat injection-extraction imbalance or ground temperature drift in the objective function.

KEY WORDS: MPC formulation, Objective function, Energy performance of buildings

1. INTRODUCTION

A clear scientific consensus exists on increased global warming due to anthropogenic sources that will cause disastrous effects in Earth’s ecosystems [1]. In order to mitigate the concentration of greenhouse gases (GHG) in the atmosphere, one of the latest commitments at an almost global scale is the Paris Agreement [2], which aims to limit the global warming effect to 2°C, and to a best of 1.5 °C. However, Kaya identity shows us that the level of difficulty of such a task is huge [3].

\[
CO_2 = \frac{CO_2 \cdot W \cdot GDP}{W \cdot GDP \cdot P}
\]  

where CO₂ represents the amount of carbon dioxide, W the energy use, GDP the gross domestic product and P the population of the sample. Hence, the first term of the identity represents the carbon intensity, the second is the energy intensity and the third the GDP per capita. Assuming that the GDP per capita and the population are two terms which are not likely to decrease, the task of enhancing decarbonisation of the grid and energy efficiency becomes even more challenging.

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The building sector, which accounts for almost 40% of the overall energy use [4] and one-third of the greenhouse gas emissions [5] takes a high share. Furthermore, considering that humans spend 87% of their time indoor [6], these energy use and GHG emission reductions have to be tackled without jeopardizing the occupants’ thermal comfort, which affects both their health and productivity.

Multiple measures exist to reduce energy use and GHG emissions due to building heating, ventilation and air conditioning (HVAC), such as: increasing equipment efficiency, correct equipment sizing and improving equipment control. One control methodology that has gained the attention of researchers in the building sector over the last decades is Model Predictive Control (MPC), which makes explicit use of a model of the building to obtain the HVAC system control signals by solving an optimization problem that minimizes an objective function, also known as cost function [7].

1.1 MPC and hybrid GEOTABS buildings  MPC is an optimal controller which uses weather forecast and a model of the building (called controller model) to predict the building needs and to optimize the control actions accordingly [8]. This control methodology is particularly interesting in (hybrid) GEOTABS buildings – i.e., buildings whose heat/cold supply system includes a geothermal heat pump and borefield (GEO) and whose emission system involves thermally activating the building system (TABS) by means of concrete core activation (CCA) – since MPC is able to anticipate their high thermal inertia and thus harness their storage capabilities [9]. Nonetheless, to cope with the slow-reacting nature of TABS and borefields, a fast-reacting secondary supply and/or emission system is often installed, leading to the hybrid GEOTABS concept [10]. The augmented complexity of such buildings with multiple (interacting) components enforces and motivates even more the necessity of MPC. Hence, a correct optimal control problem (OCP) formulation that takes into account the different components is of utmost importance to attain a desirable system behavior.

1.2 MPC formulation  MPC applied to buildings is usually based on multi-objective optimization, which involves two or more objective functions in the OCP formulation. In multi-objective optimization, the terms of the objective function are generally conflicting (e.g., energy use of the building and thermal discomfort of the occupants) and they are often adjusted with weighting factors to obtain a summed weighted objective function (see Equation (2)) . Thus, weighting factors implicitly give more priority to either the one or the other term and their adjustment becomes one of the important ingredients to achieve appropriate results. However, other approaches than weighting factors exist, such as lexicographic MPC, where one of the objectives is optimized in a prioritized way [11]. The formulation is subjected to constraints (Equation (3)), typically related to thermal comfort requirements and power limits of the components.

\[
J = \sum_{i=1}^{k=2} J_i = \alpha_1 J_1 + \alpha_2 J_2 + \cdots + \alpha_k J_k
\]

s.t.

\[
G_j \leq 0, \ j = 1, 2, 3 \ldots m
\]

The cost function minimization can cover everything that can be quantified in a mathematical way within the model, hence it is a key feature to obtain the desired results. Energy use and thermal discomfort are the most common objectives to be minimized, but other objectives could be optimized as well, such as monetary costs, GHG emissions, use of renewable energy sources (RES), flexibility and demand response indicators, etc… Furthermore, several indicators exist to measure thermal comfort and indoor environmental quality (IEQ). Moreover, in practical implementations, extra terms could be included to improve robustness of the OCP. This paper gives a review of building MPC formulations and is structured as follows: Section 2 discusses an OCP for users that prefer monetary savings. In contrast, Section 3 approaches the OCP from green users’ point of view. Section 4 explains how to handle thermal comfort and IEQ in the MPC formulation. Section 5 analyzes how to improve the robustness of the MPC in the OCP formulation. Conclusions and future work are summarized in Section 6.
2. ECONOMICAL MPC

The current structure of the final electricity price consists of multiple contributions that can be divided into fixed items and items related to the amount of energy used. The ratio between these two items depends on various factors, among them: voltage level, end consumer, yearly electricity consumption - the smaller the consumption and circuit breaker, the higher the fixed item payments.

Fixed items consist of a payment for reserved capacity based on the current main circuit breaker installed before the meter. The items related to the energy used can be split into a regulated part, which covers an electricity tax, the monthly fee, electricity transport and distribution fee, and the variable part that represents payment for the actual electricity consumption and additional services including fees for using the electricity grid. So in total, the price for the electricity commodity can represent up to 30-50% of total costs of electricity.

With the mass implementation of smart meters, it becomes possible for end customers to select from a wider variety of electricity commodity payment options. Typically, the flat price or low-high tariff options were used in the past (and still today in many countries). Nowadays, it becomes possible to use hourly energy prices or in some EU countries even 15-minutes energy prices, which can be obtained from the energy supplier or aggregator companies. A wide variety of options exists.

Intra-day wholesale market prices, which change from hour to hour (in some countries every 15 minutes), show the highest variability. A comparison with the daily electricity market is represented in Fig. 1. Note that the price signals have the same scale, however for the sake of trade secret, the absolute values are not included.

![Fig. 1 Difference between prices on daily market (DM) and intra-day market (IDM).](image)

The (discrete) formulation of the MPC problem is presented by Equation (3). Note that we include the energy use term $J_e$ only, the comfort term $J_c$ is analyzed in Section 4. The idea behind this formulation is that the heat pump (HP) power is controlled based on the electricity price signal. For hybrid GEOTABS buildings, one important feature to be included is the possibility of passive cooling (PC). Furthermore, if the secondary production system of the building is a traditional oil-, gas- or biomass-fired boiler, the associated cost term must be adapted to include the monetary cost associated to the use of oil, gas or wood. These combustibles have also time variant prices and are typically stored in storage tanks, however gas is usually supplied by the gas grid.

\[
\min \sum_{k=0}^{N-1} J_e = \min \sum_{k=0}^{N-1} \left[ c_{el}(k) \frac{Q_{HP}(k)}{\text{COP}(k)} + c_{el}(k) \frac{Q_{PC}(k)}{\text{COP}(k)} + \sum_i c_{SS}(k)_i \frac{Q_{SS}(k)_i}{\text{SS}(k)_i} \right] \Delta t \tag{4}
\]

s.t.

\[
0 \leq Q_{HP}(k) \leq Q_{HP,max} \tag{5}
\]

\[
0 \leq Q_{PC}(k) \leq Q_{PC,max} \tag{5}
\]

\[
0 \leq Q_{SS}(k)_i \leq Q_{SS,max,i} \tag{5}
\]

3
The symbol $k$ represents the time-step of the controller. The cost function to be minimized contains a weighted sum of the heat and/or cold produced by the different supply systems over the prediction horizon of length $N$. The weighting factor is composed of the supply system efficiency and the cost of the energy vector used $c$, which corresponds to the price signal forecast obtained from the different suppliers. The $COP$ is the coefficient of performance of the HP and it is a time varying parameter that depends on various quantities, but in this study we assume only dependency on ambient temperature. Notice that if the heat pump is reversible, the $COP$ would be substituted by the $EER$ when operating in cooling mode. The second term refers to the passive cooling mode, of which the efficiency depends mainly on the temperature of the soil. If passive cooling is continuously applied, the temperature of the soil will increase to a point where passive cooling is no longer possible. The last term includes the sum of all secondary supply systems in the building, if more than one. All these terms are subjected to power limiting constraints (Equation (5)), depending on the component being used.

2.1 Demand response Power from RES is highly variable and unpredictable, which can lead to unforeseen peaks that may cause instability and congestion of the electricity grid, ultimately leading to RES curtailment. Integration of RES into the electrical distribution grid comes thus along with higher requirements on control of the supply side. As the amount of electricity produced from RES has been growing significantly in EU in recent years, it becomes evident that the electricity grid stability cannot be achieved only by appropriate control on the production side, however active participation of end electricity consumers is also required. The active participation is usually achieved by so called demand-side management (DSM) that includes both demand response (DR) and energy efficiency. The reasons for actions taken by DSM are versatile, namely: i) avoiding RES power curtailment, ii) maximizing auto consumption, iii) minimizing procurement cost of electricity, iv) minimizing imbalance costs or cost of ancillary services...

The proposed economical MPC improves stability of the electricity grid as the system uses electricity mainly when there is a power surplus in the grid (which leads to lower costs). As such the energy is delivered in a cost-optimal way within both time and availability in the grid. Moreover, while the MPC drives the customer to use primarily the cheapest energy on the market, the provider saves money by having information about the amount of required energy during the next period at hand. If the grid operator asks to limit the electricity use, one way to proceed would be to include a variable constraint for the sum of the maximum power of the electricity-based supply systems. The MPC would then use the aforementioned predictions to shift the load to harness the thermal mass of the building. Demand response programs can earn back up to 15% of the electricity bill [12]. To exploit this potential demand response systems (DRS) should be set up to: i) remotely control electrical loads and ii) effectively use batteries and thermal energy storage. Heat pumps can play an important role in this context as they can be controlled in order to achieve load shifting or peak shaving. The energy storage capabilities of GEOTABS buildings make them important players. Furthermore, non-electrical based secondary systems available in hybrid GEOTABS buildings present an extra degree of freedom.

3. MPC MINIMIZING GHG EMISSIONS

The price profile does not necessarily coincide with the GHG emissions profile, as shown by Fig. 2a. While the former is dependent mainly on the electricity supply and demand, the GHG emission factor varies with the generation systems active at the moment considered. In Fig. 2b we can see that the peaks in the electricity generation (green) are approximately the same, in contrast to what happens in the CO$_2$ emissions profile (orange in Fig. 2a). On this particular case, this was caused due to a major availability of wind energy on the 25th of January. Thus, minimization of the operational costs of heating and cooling systems does not lead automatically to the lowest GHG emissions, while the latter is one of the principal objectives of the environmental policies developed by the different countries.

The minimal GHG emission MPC formulation is similar to the economic MPC formulation with time varying electricity prices, but the prices are replaced by emission factors $e$ that can be provided or estimated through
generation schedules by the grid operators (e.g. Elia in Belgium, Red Eléctrica Española in Spain or ČEPS in Czech Republic). These emission factors can change the way a hybrid GEOTABS building anticipates the disturbances and harnesses the thermal inertia of the building and the borefield. Several setups are possible, which differ in the complexity of the formulation.

![Image of graphs comparing electricity prices and CO₂ emissions](image)

**Fig. 2** (a) Comparison between electricity prices (regulated market) and CO₂ emissions associated to electricity generation and (b) electricity generation between 24/01/2018 and 25/01/2018 in Spain. Data extracted from [13].

\[
\min \sum_{k=0}^{N-1} e_{CO2,el}(k) \frac{Q_{HP}(k)}{COP(k)} + e_{CO2,el}(k) \frac{Q_{PC}(k)}{\eta_{PC}(k)} + \sum_i e_{CO2,SS}(k)i \frac{Q_{SS}(k)i}{\eta_{SS}(k)i} \Delta t
\]

s.t.

\[0 \leq Q_{HP}(k) \leq Q_{HP,\text{max}}\]

\[0 \leq Q_{PC}(k) \leq Q_{PC,\text{max}}\]

\[0 \leq Q_{SS}(k)i \leq Q_{SS,\text{max},i}\]

### 3.1 Building supplied by green energy
In this case, the building owner or tenants have a contract with an electricity supplier who guarantees that electricity will be supplied from RES (PV, wind farms, water power plants, etc…). In this case, there is nothing to optimize because RES has zero GHG emission.

### 3.2 Building without local RES, no green energy from grid
Here we consider the case where the electricity supplier delivers electricity from the grid without the guarantee that it originates from RES, and the building has no local electricity production from RES. Then it is important to take into account the emission factors. In general, the emission factors for the specific location are time varying – e.g., the actual value of the emission factor will differ between summer and winter if a lot of PV electricity is injected in the grid.

### 3.3 Building with local RES, no green energy from grid
This case has the highest complexity, since it is important to take into account both the forecast of other electricity consumers and the electricity production by local RES (PV, wind, etc…). If the local RES produce more electricity than needed by the building (heat pump and other consumptions), then the carbon footprint is zero. If the production of local RES is not sufficient, then some electricity must be obtained from the grid and the correct emission factor has to be taken into account.

Fig. 3 depicts the cost function for optimization. Note that this type of cost functions can be formulated and optimized with the aid of slack variables.
Fig. 3 Cost function trend: \( f_1(k) = \frac{Q_{HP}(k)}{COP(k)} + P_{\text{other}}(k) - P_{\text{local, prod}}(k) \), \( f_2 \) cost function, for \( f_1(k) \geq 0 \), \( f_2(k) = e_{\text{CO}_2}(k)f_1(k) \), otherwise \( f_2(k) = 0 \)

3.4 Building with conventional fossil energy source heating systems If the secondary system of the building is a traditional oil- or gas-fired boiler, an additional term should be added to the objective function that takes into account the GHG emissions of oil or gas combustion. Typical values for these emission systems can be found at Table 1. For biomass boilers it is assumed that the net balance of \( \text{CO}_2 \) emission is zero.

<table>
<thead>
<tr>
<th>Combustible</th>
<th>( \text{CO}_2 ) emission, low CP [kg/kWh]</th>
<th>( \text{CO}_2 ) emission, high CP [kg/kWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Gas</td>
<td>0.181</td>
<td>0.200</td>
</tr>
<tr>
<td>Fuel oil</td>
<td>0.247</td>
<td>0.263</td>
</tr>
<tr>
<td>Wood</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4. THERMAL COMFORT AND INDOOR ENVIRONMENTAL QUALITY

The main purpose of designing heating, cooling and ventilation systems in buildings is to achieve a minimum level of thermal comfort and indoor air quality (IAQ) for the occupants. Enhanced indoor environmental quality (IEQ) can improve occupants productivity by 5 to 10% [15], which may be a significant cost saving especially in office buildings. Furthermore, elderly people prefer warmer thermal conditions [16], a factor to take into account in elderly care homes. Thus, it is clear that this aspect has to be included somehow in the OCP. In sections 2 and 3, we have analyzed the term corresponding to energy use \( J_c \) without taking into account thermal comfort \( J_t \). Some MPC formulations [17] have included the latter as temperature bounds within hard constraints, however this formulation could lead to unfeasibility issues, which need to be tackled by the introduction of slack variables. Moreover, if slack variables are used to track a determined set-point, this would limit the freedom of the MPC and may result in higher energy use [18]. Therefore, temperature bounds are desirable combined with a penalization for crossing the bounds.

Several thermal comfort standards exist to define the upper and lower temperature (and other comfort parameters) bounds of a building, such as ISO7730, EN15251, ASHRAE55 and ISO74, extensively discussed by Sourbron and Helsen [19]. These models are either based on thermal comfort bounds or on the PMV model of Fanger [20]. However, the non-linear nature of the latter makes it computationally more expensive, leading to the use of simplified versions of this model [21]. These are not the only thermal comfort models found in the literature, for more details the reader is referred to Enescu [22]. Some studies recommend an adaptive thermal model that involves acclimation of people, which improves people’s health by increasing their thermo-neutral zone [23].
Moreover, appropriate thermal comfort does not ensure a good IEQ since this depends on additional factors, such as indoor air quality (IAQ), lighting quality, visual and acoustic comfort… We focus on IAQ to improve the overall IEQ, which is usually enhanced by ventilation strategies. New evidence exists that mechanical ventilation systems lead to an overall improvement of the IAQ and reduction of reported comfort and health related problems [24]. If the building is equipped with an air handling unit and CO₂ sensors, efficient control can contribute to enhanced IAQ. However, MPC needs an occupancy model to predict the ventilation needs, e.g. based on statistical data or on available measurements [25]. These occupancy models are also important to predict thermal loads and thus improve thermal comfort (in the end, humans are walking radiators), and when correctly implemented they can further save up to 30% energy [26]. The proposed formulation includes therefore a slack term for thermal comfort \( s_T \) and another for IAQ comfort \( s_{CO2} \). \( \alpha_T \) and \( \alpha_{CO2} \) are the weighting factors that represent the “price” the final user is willing to pay to have more or less comfort, \( l_b \) and \( u_b \) represent the lower and upper bound for the chosen thermal comfort model and CO₂ levels.

\[
\min \sum_{k=0}^{N-1} j_c = \min \sum_{k=0}^{N-1} [\alpha_T(k) s_T(k) + \alpha_{CO2}(k)s_{CO2}(k)] \Delta t \\
\text{s.t.} \quad l_b, T_s + s_T \leq T_{zones} \leq u_b, T_s + s_T \\
\quad \quad l_b, CO2 + s_{CO2} \leq CO_{2zones} \leq u_b, CO2 + s_{CO2}
\]

The hybrid GEOTABS concept can improve both thermal comfort and IEQ. TABS can provide an ideal vertical temperature gradient, and due to the small temperature differences between the surfaces and the space, the system can benefit from the self-regulating effect and provide a stable thermal environment [27]. Buildings with mechanical ventilation units can use these as the fast-reacting secondary system by pre-heating or pre-cooling the air before being injected in the building zones. The presence of TABS significantly reduces the size of the ventilation system (and corresponding fan power) to provide acceptable IAQ if the necessary heating or cooling at peak times. As a consequence, IEQ is also improved: less draught and noise from fans, no visible heating/cooling devices…

5. ADDITIONAL ROBUSTNESS

Perfect predictions would lead to a smooth behavior of the MPC. However, in real implementations MPC has to deal with several uncertainties, i.e. accuracy of predictions, measurement errors, model mismatch… Additional features to improve MPC robustness can be included in the OCP formulation. One example has already been mentioned in section 4: thermal comfort bounds are included as slack variables in the objective function to avoid unfeasibility problems.

Another problem that can appear is oscillatory behavior. If the constraints are not very tight, the control actions result into either idle (no energy) or deadbeat control (full power), thus in control actions that need post-processing. This behavior causes issues, especially in closed-loop performance, where the control actions can have a very oscillatory behavior. These oscillations can be eliminated by introducing constraints in the rate of change of the delivered energy to the building [18]. The introduced constraints should be soft constraints to avoid problems with cases were full power is really required (e.g., after a long holiday period). Terms such as minimizing the maximum rate of change or the curvature of the delivered inputs can be included in the objective function.

MPC predicts over a chosen prediction horizon, which cannot be taken too long (maximum in the order of weeks) since this would lead to a too high number of optimization variables. As a consequence it is difficult to incorporate in the MPC the effect of seasonal energy storage in the borefield. However, to avoid thermal depletion of the borefield, a thermal balance in the ground should be ensured on the long term. To this end, some authors [9] have included a long-term cost in the objective function, that penalizes the use of the borefield
at specific moments thereby inviting the system to use the secondary production unit. The thermal conductivity of the ground plays a crucial role in this thermal ground balance. For grounds with low thermal conductivity additional exploitation of seasonal thermal energy storage in the borefield may become economically beneficial. This switching point depends on the efficiency of the secondary (heat/cold) production units in relation to the heat pump and passive cooling COP. Storing energy always leads to losses [28].

Design of hybrid GEOTABS systems is often based on static methods (described in standards). However both TABS and borefield are usually in transient states due to their large thermal inertia. Therefore, using a dynamic controller model in the MPC is very important.

6. CONCLUSIONS AND FUTURE RESEARCH

Several OCP formulations have been proposed based on the hybrid GEOTABS buildings properties and literature review. Most of the formulations include multi-objective optimization based on the trade-off between energy use and thermal comfort. However, the way these terms are weighted is diverse and should be adapted to the final user needs. Energy use can be converted to energy cost by using price profiles or converted to GHG emissions by using CO$_2$ generation profiles. Thermal comfort can be adapted to satisfy the user’s subjective comfort and enhance overall IEQ. Robustness of the MPC can also be increased by incorporating additional terms that remove oscillatory behavior and ensure thermal balance of the ground.

Future research includes testing the proposed formulations in virtual test beds and analyzing the influence of diverse parameters – e.g., the prediction horizon, cost profile – to check how MPC optimizes and harnesses the buffering capabilities of the hybrid GEOTABS buildings. Diverse thermal comfort models will also be tested to find out which one fits best to the hybrid GEOTABS concept. Robustness terms will be included to mitigate unwanted oscillations and guarantee thermal balance of the ground.

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REFERENCES


